Chemistry
Uncertainty & Sig. figs.

Appendix 5

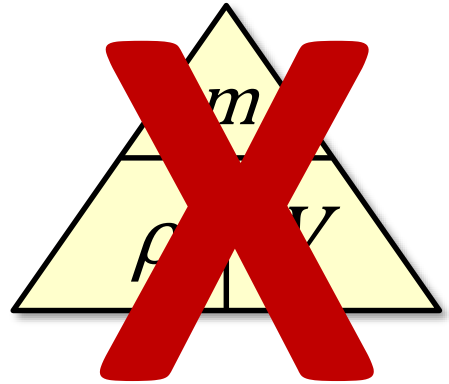
“Quantum Chemistry”

Density

Density is the volume/mass relationship of an object.

$$\boxed{\rho = \frac{m}{V}}$$

or



Where: $\rho =$ the density of the object, in g/cm^3 or g/mL

m = the mass of the object, in g

V = the volume of the object, in cm^3 or mL

$$\boxed{\rho_W = 1\ {}^{\mathrm{g}}/_{\mathrm{mL}} = 1\ {}^{\mathrm{g}}/_{\mathrm{cm}^3}}$$

The density of water is 1 g/mL.
This is not true of other substances.
Objects with less density than water will float. Objects with greater density will sink.

Solving Problems

- When solving Chemistry problems on a test or exam, it is important not only to find the correct answer, but to **justify it**.

1. Show your **data**,
2. Show your work, including the **formulas** you used and the **substitutions** you made.
3. Write an **answer statement**, a sentence that clearly states your final answer.
4. Include the correct **units** for your answer. Never just give a number—you must specify what the number means!

Suggested Solution Method

Problem: A block of material has a length of 12.0 cm, a width of 5.0 cm, and a height of 2.0 cm. Its mass is 50.0 g.

Find its density.

List all the information you find in the problem, complete with units, and the symbols.	*Show the substitutions you make, and enough of your calculations to justify your solution:*
Data: l = 12 cm. w = 5.0 cm h =2.0 cm m =50g V = ? To Find: ρ (density)	Calculations: $V = lwh$ $= 12cm \times 5cm \times 2\ cm$ $= 120\ cm^3$ $\rho = \frac{m}{V}$ $= 50\ g / 120\ cm^3$ $= 0.42\ g/cm^3$

Answer: The density of the block is 0.42 g/cm³.

App.5
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Overview: Uncertainty

The Inherent Errors of Instruments

- All measurements are made using instruments, and all instruments have **imperfections** that limit their precision.
- This is often called the “**error**” or “**uncertainty**” of the instrument.
- This is **not a mistake** made by the observer, but rather an unavoidable reality of an imperfect world.
- When we make measurements you must record the error/uncertainty involved.

Absolute Uncertainty (AU)

In math:
they all represent perfect number 2.

2 mL
2.0 mL
2.00 mL

In science:
they all have a different degree of of uncertainty.

All instruments that we use to make measurements have an inherent error or *absolute uncertainty*.

Notes! Please write!

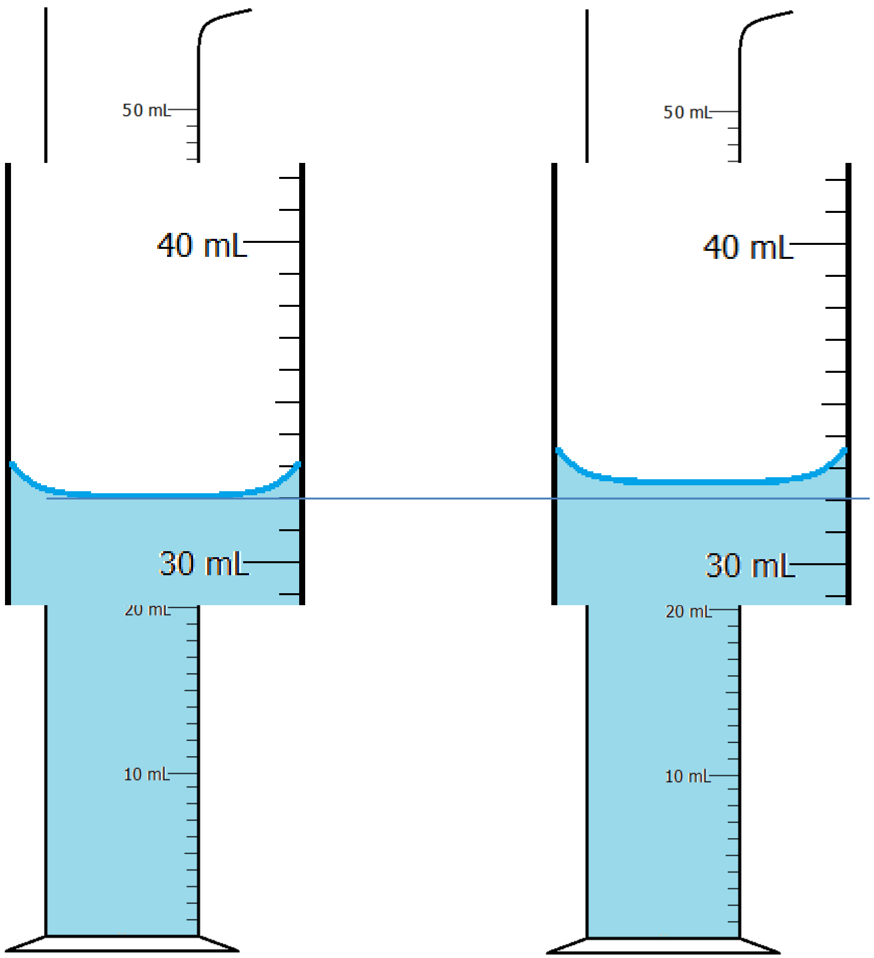
Absolute uncertainty (AU)

- is the error range of an instrument. Unless stated otherwise we will assume it is ½ the measure between the smallest markings.

  eg. (32.5 $\pm$ 0.5) mL

- The 0.5 tells you that you must read the measurement to one decimal place.

Example of Uncertainty.



- At first glance, the two graduated cylinders here seem identical, but look closer.
- The first one has a measurement of 32.0 ± 0.5 mL
- The second one 32.5 ± 0.5 mL
- It is NOT correct to say that the first measurement is just 32 mL!

How to Record Absolute Uncertainty

Plus-minus sign

Unit

(32.5 ±0.5)mL

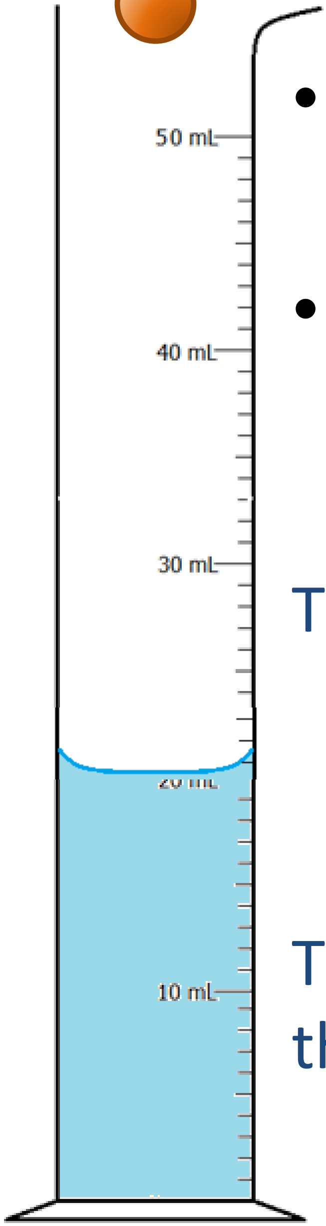
A pair of parenthesis <u>may</u> be placed around the measurement.

Doubtful, but Significant digit

Absolute Uncertainty

1. When you first look at the second graduated cylinder, it appears to contain 32 mL of liquid.
2. Looking closer, you see it is about halfway between 32 and 33 mL, so you record the **.5**
3. Then you write the **absolute uncertainty**, the error range.

Adding & Subtracting with Absolute Uncertainties



- Frequently we make two measurements and subtract them to find a difference (*Δ*).
- When we subtract numbers that have an uncertainty we must <u>ADD</u> the absolute uncertainty values!

The volume changes from (20.5±0.5)mL to (24.0±0.5)mL.

$$AU_T = \Sigma AU \qquad \text{or} \qquad AU_T = AU_1 + AU_2 + \ldots$$

The volume difference (*Δ*V) is (3.5±1.0)mL but because the uncertainty is larger we write (4 ± 1)mL.

Absolute uncertainty (AU) **Notes! Please write!**

- is the allowable error of the instrument. Unless stated otherwise we will assume it is ½ the measure between the smallest markings.

  eg. (32.5 ± 0.5) mL

- The 0.5 tells you that you must read the measurement to one decimal place.

- When adding or subtracting #s with uncertainties, we must add the uncertainties.

  Eg. $(24.0 \pm 0.5)\text{mL} - (20.5 \pm 0.5)\text{mL} = (3.5 \pm 1.0)\text{mL}$

  $= (4 \pm 1)\text{mL}$

- The AU is always rounded to one significant digit!

WS1

Due next class

Relative Uncertainty

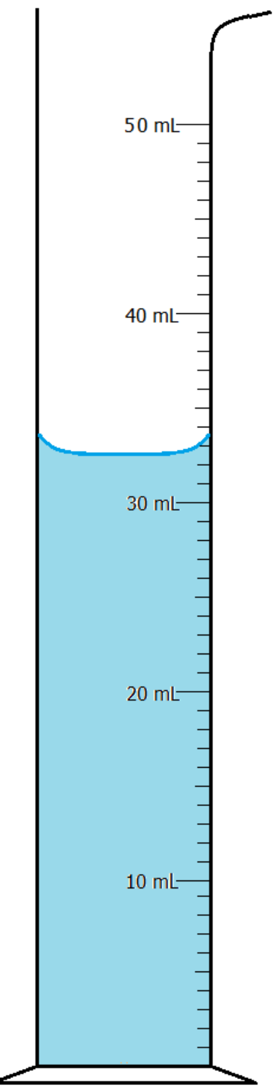
- Sometimes it is useful to know how much uncertainty we have compared to the original measurement.
- To do this we can calculate the relative uncertainty (RU).
- RU of a measurement equals the absolute uncertainty divided by the absolute value of the original measurement.

$$\boxed{RU = \frac{AU}{|measurement|}}$$

- The resulting decimal number is often converted to a percentage

Click here for details on UNCERTAINTY.

Example of Relative Uncertainty



- The graduated cylinder has a reading of (32.5±0.5) mL
- To find its relative uncertainty, divide:

  $0.5 \div 32.5 = 0.01538461$

- Round off to a reasonable number of decimal places and convert to a percent:

  $0.015 \times 100 = 1.5\%$

- Write it like this:

  32.5 ml ±1.5%

The nice thing about relative uncertainties it that they show you how small your error actually is.

Notes! Please write!

Relative uncertainty (RU)

- RU of a measurement equals the absolute uncertainty divided by the absolute value of the original measurement.

$$RU = \frac{AU}{|\text{measurement}|}$$

eg. 32.5 $\pm$ 0.5 mL

RU = 0.5/32.5 = 0.0153846... Or 1.5%

- The RU allows you to determine the AU after a calculation involving multiplication or division.

Multiplying & Dividing with Uncertainties

Density example: (58.3±0.9)g ÷ (32.5±0.5)mL.

- Step 1: Do the calculation: 58.3 ÷ 32.5 = 1.79385

  (keep 4-6 digits!)

- Step 2: add the RU values.

  (0.9/58.3 + 0.5/32.5) = ? 0.030822

- Step 3: multiply the RU with your answer:

  (0.9/58.3 + 0.5/32.5) x 1.79385

  = 0.0552901

Step 4: round to ONE significant digit.

The new AU is 0.06!

Final answer: (1.79 ± 0.06) g/mL

Notes! Please write!

Density example: $(58.3 \pm 0.9)g \div (32.5 \pm 0.5)mL$.

$$P = \frac{m}{V}$$

$$= \frac{58.3}{32.5}$$

$$P = 1.79385$$

$$\delta P = \left(\frac{\delta m}{m} + \frac{\delta V}{V}\right) P$$

$$= \left(\frac{0.9}{58.3} + \frac{0.5}{32.5}\right) \times 1.79385$$

$\delta P = 0.0552901$ round to one significant digit

$\delta P = 0.06$

Therefore you must round to two decimal places!

Final answer: (1.79 ± 0.06) g/mL

Rule: (A ± ΔA) + (B ± ΔB) = (A + B) ± (ΔA + ΔB)

(A ± ΔA) - (B ± ΔB) = (A - B) ± (ΔA + ΔB)

Rule: (A ± εA) x (B ± εB) = (A x B) ± (εA + εB)

(A ± εA) / (B ± εB) = (A / B) ± (εA + εB)

$$\delta(\Delta L) = \sqrt{(\delta L1)^2 + (\delta L2)^2}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(X_i - X)^2}{N-1}} = \sqrt{\frac{(X_1 - X)^2 + (X_2 - X)^2 + \ldots + (X_N - X)^2}{N-1}}$$

Uncertainty Formulas

Please write!

Adding or subtracting: add the uncertainties

$$(A + B) \pm (\delta A + \delta B)$$

$$(A - B) \pm (\delta A + \delta B)$$

Multiplying or dividing: add the relative uncertainties x the solution

$$\delta AB = \left(\frac{\delta A}{A} + \frac{\delta B}{B}\right) AB$$

$$\delta\left(\frac{A}{B}\right) = \left(\frac{\delta A}{A} + \frac{\delta B}{B}\right) \frac{A}{B}$$

WS2 AU & RU calculations

Due next class

Notes! Please write!

Significant Figures (or digits):

- They are a guide to how much we should round off a calculated answer.
- This is useful when you are doing calculation problems where the uncertainty is not provided, but instead needs to be inferred by the #s given!

Notes! Please write!

Rules for Significant Figures :

1. All non-zero digits are **ALWAYS** significant
2. Zeros between significant digits are **ALWAYS** significant.
3. Zeros at the beginning of a number are **NEVER** significant.
4. Zeros at the end of a number **MAY** be significant.
5. Exponents, multiples, signs, etc. are **NEVER** significant.

Examples of Rule 1, 2 and 3

Rule 1. Non-zero digits are **ALWAYS** significant.

1.234

145

19567.2

Rule 2. Zeros between significant digits **ARE** significant.

1001

5007.4

20000.6

Rule 3. Zeros at the beginning are **NEVER** significant.

007

0.0000005

0.025

Explaining Rule 4

Rule 4. Zeros at the end of a number **MAY** be significant.

Your textbook says that they are **ALWAYS** significant, but this is contrary to what most textbooks say.

If there is a **decimal point**, there is no problem. All textbooks agree, the

In science we should write the #s in scientific notation.

Only significant #s are written!

For this class ….

You can assume all #s in a problem are significant.

250	has 2 or 3 significant figures
123 000 000	has 3 to 9 significant figures

In a test situation, assume the numbers are precise, unless something in the question states otherwise.

Estimated source

Trusted precise source

Rule 5

Rule 5: Exponents and their bases, perfect multiples, uncertainties (error values), signs etc. are **NEVER** significant.

6.02 x 10^{23} has 3 significant digits

504.1 mL x 3 has 4 significant digits (the 3 is not measured)

5.3 ±0.5 mL has 2 significant digits

– **5.432** x 10^{-5} has 4 significant digits

π x **3.54** cm has 3 significant digits (π is not a measurement)

In each case,

the **blue** part is significant, the green part is **NOT** significant.

Notes! Please write!

Rules for Significant Figures :

1. All non-zero digits are **ALWAYS** significant

   21 = 2 sig. figs

2. Zeros between significant digits are **ALWAYS** significant.

   2001 = 4 sig. figs

3. Zeros at the beginning of a number are **NEVER** significant.

   0.04 = 1 sig. fig.

4. Zeros at the end of a number **MAY** be significant.

   21.0 = 3 sig. figs

5. Exponents, multiples, signs, absolute errors etc. are **NEVER** significant.

Math with Significant Figures

- Adding and Subtracting:
  - All units must be the same (can't add different units!)
  - Line up all the measurements at their decimal points.
  - Add or subtract as normal.
  - Round off all numbers to match the shortest number of decimals.

Decimals lined up

Round off

Example: add the following measurements.

This unit is not the same as the others!
(litres vs. millilitres)

~~5.34576 L~~ → 5345.7 6 mL

55.1 43mL → 55.1 43mL

547.1 mL → 547.1 mL

5948.0 03 mL

The answer is 5948.0 mL. Note that the answer has 5 sig. digits, even though one of the measurements had only 4 sig. digits. .

- Multiplication and Division:

  - Different units may be multiplied or divided if there is a formula to justify it.
  - The main rule in multiplying and dividing is that you cannot have an answer with more significant digits than your "weakest" measurement (the one with fewest significant digits)
  - After doing the math, round off your answer to match the weakest measurement.

Weakest measurement **only 3 Sif.Figs.**

Multiply 2.53 g/mL by 75.35 mL

2.53 x 75.35 = 190.6355

=191 g

Answer has 3 Sig.Figs.

Justification:
$m = \rho V$

About the unit:
$\frac{g}{\cancel{mL}} \times \cancel{mL} = g$

You are the weakest link. Goodbye!

Math with Significant Figures

- Perfect numbers
  - Double a quantity → the 2 you multiply by is considered perfect.
  - Mole ratios in stoichiometry
  - π
- Generally, use the same rule as for multiplying for square roots, exponents etc.
  - That is, your answer can have no more significant digits than your weakest measurement.
- Mixed operations
  - Follow BEDMAS

Notes! Please write!

- Adding and Subtracting:
  - All units must be the same (can’t add different units!)
  - Line up all the measurements at their decimal points.
  - Add or subtract as normal.
  - Round off all numbers to match the fewest # of decimals.
- Multiplication and Division:
  - After doing the math, round off your answer to match the lowest # of sig.figs.
- Combination
  - Follow BEDMAS

- WS3

- Significant figures

Problems on Significant Figures

1. How many significant digits are in each measurement:
   a) 123.45 mL
   b) 4.500×10^3 mL
   c) 7 apples
   d) times 5
   e) 0.0023 m
   f) 4000 kg
2. A Coulter counter is a device which counts the blood cells in a sample as they pass through a beam of light. A laboratory technician records 20000 wbc in a blood sample. At a demonstration a reporter says there were 20000 protesters. Both numbers are the same, which one has more significant figures? Why?
3. Find the volume of a cube that measures 2.3 cm by 3.55 cm by 2.14159 cm.
4. Add the measurements: 2.500 kg, 354.2 g, 153.78 g

Also: Do the worksheet entitled “Significant Figures”

Density of some metals

Metal	Density (g/cm^3)
Aluminum	2.70
Copper	8.95
Gold	19.3
Iron	7.87
Lead	11.3
Magnesium	1.74
Silver	10.5
Tin	7.26
zinc	7.14